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The spectrum of $N = 3$ string theory on $AdS_3 \times G/H$

Riccardo Argurio

Dipartimento di Fisica, Università Milano Bicocca
Piazza delle Scienze 3, Milano, Italy
E-mail: Riccardo.Argurio@mi.infn.it

Amit Giveon and Assaf Shomer

Racah Institute of Physics, The Hebrew University
Jerusalem 91904, Israel
E-mail: giveon@vms.huji.ac.il, shomer@cc.huji.ac.il

Abstract: We continue the study of string theory on $AdS_3 \times \mathrm{SU}(3)/\mathrm{U}(1)$ and $AdS_3 \times \mathrm{SO}(5)/\mathrm{SO}(3)$. We compute the spacetime spectrum of the $N = 3$ supersymmetric dual CFT using worldsheet techniques. The spectrum of chiral primaries coincides for the two models. Unlike $N = 4$ theories, the building block of the symmetric product in spacetime (corresponding to a single long string) is not by itself in the moduli space of a symmetric product.

Keywords: AdS/CFT Correspondance, Conformal Field Models in String Theory, Extended Supersymmetry, Superstring Vacua.

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Contents

1. Introduction

In this paper we continue the study of superstring theory on the backgrounds $AdS_3 \times \mathrm{SU}(3)/\mathrm{U}(1)$ and $AdS_3 \times \mathrm{SO}(5)/\mathrm{SO}(3)$. In [1] we showed that superstrings propagating on these backgrounds give rise to a dual CFT with $N = 3$ superconformal symmetry (or $N = 1$, depending on the GSO projection). Following the general result of [2] the spacetime dual CFT is expected to be in the moduli space of a symmetric product CFT $(\mathcal{M}_{6k})^p/S_p$ with p the maximal number of long strings discussed in [2]–[7]. Here we will compute the chiral spectrum of $\mathcal{M}_{6k}$ (i.e. the untwisted sector of the dual CFT in the language of [2]) using perturbative worldsheet analysis. This will enable us to draw some conclusions about the nature of the spacetime CFT that were also mentioned in [2]. In particular, unlike other cases [3, 8] which have a larger ($N = 4$) symmetry, the building block $\mathcal{M}_{6k}$ is not in the moduli space of any symmetric product.

1.1 A short review of superstrings on $AdS_3 \times G/H$

We start by reviewing the formulation of superstrings on AdS_3 times a coset CFT, following mainly the formalism of [3, 9] and, for simplicity, treating only the holomorphic sector. We begin by setting some notation.

The AdS_3 part is described by an SL(2) supersymmetric WZW model that is constituted of the three currents J^P of the SL(2) affine algebra at level k, and the three fermions ψ^P implied by the $N = 1$ worldsheet supersymmetry, satisfying the following OPEs:

$$\begin{aligned} J^P(z)J^Q(w) &\sim \frac{\frac{k}{2}\eta^{PQ}}{(z-w)^2} + \frac{i\epsilon^{PQR}\eta_{RS}J^S(w)}{z-w}\,, \\ J^P(z)\psi^Q(w) &\sim \frac{i\epsilon^{PQR}\eta_{RS}\psi^S(w)}{z-w}\,, \\ \psi^P(z)\psi^Q(w) &\sim \frac{\frac{k}{2}\eta^{PQ}}{z-w}\,, \end{aligned} \tag{1.1}$$

where $P, Q, R, S = 1, 2, 3$, $\eta^{PQ} = (+ + -)$ and $\epsilon^{123} = 1$.

As usual in supersymmetric WZW models, the currents can be decomposed in two pieces:

$$J^P = \hat{J}^P - \frac{i}{k}\eta^{PQ}\epsilon_{QRS}\psi^R\psi^S\,. \tag{1.2}$$

The first piece $\hat{J}^P$ constitutes an affine algebra at level $k+2$, and has a regular OPE with the fermions ψ^P. We will thus refer to $\hat{J}^P$ as the bosonic currents. The second part constitutes an affine algebra at level -2.

The coset CFT part is constructed by starting from the $G = \mathrm{SU}(3)$ or SO(5) super affine algebra at level $k' = 4k$ (this condition arises from criticality of the string theory [1]) realized by $\dim G$ affine currents K^A and their fermionic superpartners χ^A, satisfying the following OPEs:

$$\begin{aligned} K^A(z)K^B(w) &\sim \frac{\frac{k'}{2}\delta^{AB}}{(z-w)^2} + \frac{if_{ABC}K^C(w)}{z-w}\,, \\ K^A(z)\chi^B(w) &\sim \frac{if_{ABC}\chi^C(w)}{z-w}\,, \\ \chi^A(z)\chi^B(w) &\sim \frac{\frac{k'}{2}\delta^{AB}}{z-w}\,. \end{aligned} \tag{1.3}$$

Here $A, B, C, D = 1 \ldots \dim G$ and f_{ABC} are the structure constants of G. As before, the currents split into their bosonic and fermionic parts. The bosonic currents realize an affine algebra at level $k' - h^G$ where h^G is the dual Coxeter number of G which is the same in both cases $h^{\mathrm{SU}(3)} = h^{\mathrm{SO}(5)} = 3$. One then proceeds to construct the SU(3)/ U(1) and SO(5)/ SO(3) coset SCFTs following [9, 10].

As was shown in [1] a condition needed in order to realize an $N = 3$ algebra is that an unbroken SU(2) at level k' must remain after modding out by $H \subset G$. Such an SU(2) subgroup indeed remains in the two coset models considered here,

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and we keep the indices $A = 1, 2, 3$ for its generators and denote its spin by j'. It was then shown in [1] that these two models indeed realize an $N = 3$ superconformal symmetry in spacetime, for a definite GSO projection.

In the next section we will compute the chiral spectrum of the untwisted sector (in the sense of [2]) of the spacetime CFTs corresponding to the two coset backgrounds $AdS_3 \times \mathrm{SU}(3)/\,\mathrm{U}(1)$ and $AdS_3 \times \mathrm{SO}(5)/\,\mathrm{SO}(3)$. We therefore start with a short reminder of the chirality condition for the $N = 3$ superalgebra and of the relation between worldsheet and spacetime quantities in the general context of superstrings on AdS_3 backgrounds.

1.2 The chirality condition in the $N = 3$ superconformal algebra

The global part of the $N = 3$ superconformal algebra in the NS sector [11, 12] contains the following anti-commutator

$$\{Q^a_r, Q^b_s\} = 2\delta^{ab} L_{r+s} + i\epsilon^{ab}{}_c(r - s)T^c_0\,, \tag{1.4}$$

where $a, b, c = 1, 2, 3$ and $r, s = \pm 1/2$. We see that a unitarity bound

$$2L_0 \geq |T^3_0| \tag{1.5}$$

arises from evaluating the norm of $Q^{+(-)}_{-1/2}|h\rangle$, where $|h\rangle$ is a primary state. The chirality condition is the saturation of this bound $2L_0 = T^3_0$, i.e. that the spacetime weight equals half the spacetime R-charge which in our case arises from the unbroken SU(2) spin:

$$h_{ST} = \frac{1}{2} j_{ST}. \tag{1.6}$$

This can, of course, occur only for the highest weight state inside each SU(2) multiplet. The states with $j_{ST} = -m_{ST} = 2h_{ST}$ are called anti-chiral states and obey the opposite condition $2L_0 = -T^3_0$. Note that in the case of the $N = 3$ superconformal algebra the R-charge is quantized to be in $\frac{1}{2}\mathbb{Z}$ because the U(1) is actually the J^3 of the SU(2) R-symmetry. Therefore only the multiplicity of chiral states in each energy level is relevant.

1.3 The relation between worldsheet and spacetime quantum numbers

In [3] it was shown that vertex operators transforming under the worldsheet SL(2) algebra in the representation j have the following scaling dimension in the spacetime CFT:

$$h_{ST} = j + 1\,. \tag{1.7}$$

It was also demonstrated that an affine algebra $\hat{G}$ in the worldsheet CFT gives rise to an affine algebra in the spacetime CFT such that if the worldsheet vertex operators transform under $\hat{G}$ in some representation r^G then the corresponding spacetime fields transform in the same representation r^G under the spacetime affine currents. Since we look for chiral primary states of the spacetime CFT we have to demand two things:

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(i) The state has to be a primary field of the spacetime CFT and must therefore have a definite scaling dimension $h_{ST} = j_{\mathrm{SL}(2)} + 1$, i.e. it must transform irreducibly under the worldsheet SL(2).

(ii) The state should have a minimal spacetime weight for every representation of the spacetime superconformal R-symmetry which arises from the unbroken SU(2).

2. Computation of the chiral spectrum in spacetime

2.1 Vertex operators and the physicality condition — The NS sector

We start by considering the general form of an NS vertex operator. The construction of such vertex operators in curved backgrounds of the form $AdS_3 \times G/H$ goes along similar lines to that of the flat case [3, 13]. Here however we have to replace the plane wave zero modes with the vertex operators $\Phi_{j,m}$ and $U_{r,j',\mathcal{Q}}$ corresponding to the zero modes on AdS_3 and G/H. The vertex operators on AdS_3 are labeled by j, m according to the representation of SL(2) [3], and those on G/H by the representation of G that we denote here by r, by the spin j' of the unbroken SU(2) and possibly by other quantum numbers of G/H that are denoted here by $\mathcal{Q}$. The tower of string states is obtained as usual by multiplying these zero mode operators by a polynomial P_N of scaling dimension N in the fermionic oscillators ψ^P, χ^A and the bosonic oscillators $\hat{J}^P$, $\hat{K}^A$ and their derivatives (and a similar polynomial in the antiholomorphic oscillators). This operator has then to obey physicality conditions, usually implemented by restricting to the BRST cohomology and performing a chiral GSO projection. We thus write the general form of the NS vertex operator in the (-1) picture, writing for brevity only the holomorphic part:

$$e^{-\phi} P_N\left(\psi^P, \partial\psi^P, \chi^A, \partial\chi^A, \ldots, \hat{J}^P, \partial\hat{J}^P, \hat{K}^A, \partial\hat{K}^A\right)\Phi_{j,m}U_{r,j',\mathcal{Q}}\,. \tag{2.1}$$

The physicality condition in the case of an NS vertex operator is that it must be a primary field of weight $(1,1)$ in the worldsheet CFT (more precisely, BRST invariance is equivalent to requiring that the matter part of (2.1) is the lower component of an $N = 1$ worldsheet superfield of weight $(1/2, 1/2)$).

Considering affine algebra primaries, which are also Virasoro primaries, their weight is:

$$\Delta_G = \frac{Q^G_{r_G}}{\hat{k}_G + h^G}\,, \tag{2.2}$$

where $Q^G_{r_G}$ is the value of the quadratic Casimir operator of the group G in the representation r_G, $\hat{k}_G$ is the bosonic level of the affine G, h^G is the dual Coxeter number and we normalized to 1 the length squared of the highest root of G (see e.g. [14]).

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The weight of a primary in a coset CFT G/H is similarly obtained since the coset CFT energy-momentum tensor is $T_{G/H} = T_G - T_H$ and thus the weight is the difference $\Delta_G - \Delta_H$.

Using the above,[1] we can write the physicality condition as:

$$\frac{1}{2} + N + \frac{-j(j+1)}{k} + \frac{Q^G_{r_G} - Q^H_{r_H}}{k'} = 1\,. \tag{2.3}$$

Since we are interested in the spectrum of chiral primaries we have to look for the smallest scaling dimension in each SU(2) representation. In a similar way to the flat-space case the smallest N we can take is $N = 1/2$ because the $N = 0$ vertex is tachyonic, and will not be mutually local with the supercharges. We therefore look for vertex operators of the general form (2.1) with P_N being a linear combination of fermions such that the full vertex has a well defined SL(2) and SU(2) quantum numbers. Actually, linear combinations of $\psi^{\pm,3}$ will raise or lower the SL(2) spin, while combinations of $\chi^{1,2,3}$ will do the same for the SU(2) spin (see for instance [15, appendix A]). For the time being we concentrate on the first possibility, and consider the other at the end of this subsection.

We thus pick out the smallest SL(2) spin, i.e. the $j-1$ component $(\psi\Phi_j)_{j-1}$ which is [3, 15] (again, only the holomorphic part is shown):

$$e^{-\phi}\left(\psi^3\Phi_{j,m} - \frac{1}{2}\psi^-\Phi_{j,m+1} - \frac{1}{2}\psi^+\Phi_{j,m-1}\right)U_{r,j',\mathcal{Q}}\,, \tag{2.4}$$

and has spacetime scaling dimension $h_{ST} = j$.

Taking this into account, i.e. plugging $N = 1/2$ in (2.3) and using $k' = 4k$ we end up with the following condition:

$$j(j+1) = \frac{1}{4}\left(Q^G_{r_G} - Q^H_{r_H}\right). \tag{2.5}$$

We now turn to calculate the quadratic Casimir eigenvalues in a general representation of SU(3) and SO(5) in order to compute the chiral spectrum in the $AdS_3 \times$ SU(3)/ U(1) and $AdS_3 \times$ SO(5)/ SO(3) cases. To set our notation, we recall that given a representation $[\lambda_1, \lambda_2, \ldots, \lambda_n]$ of a Lie algebra of rank n, its highest weight vector is $\mu = \lambda_1\mu^1 + \lambda_2\mu^2 + \cdots + \lambda_n\mu^n$, where μ^i are the fundamental weights defined with respect to the simple roots α^i by $2\langle\alpha^i, \mu^j\rangle/(\alpha^i)^2 = \delta^{ij}$. The value of the quadratic Casimir of the above representation is then given by the inner product in weight-space, $Q = \langle\mu, \mu+\rho\rangle$, where $\rho = \sum\alpha^+$ is the sum of all positive roots.

[1]Note that the total levels of the affine G and H are the same, because the embedding $H \subset G$ is such and $k' = \hat{k}_G + h^G = \hat{k}_H + h^H$, and in both cases of interest the highest root of H has the same length as the highest root of G, so that the normalization in Δ_G and Δ_H does not change.

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2.2 The SU(3)/ U(1) coset CFT

The SU(3) algebra is simply laced, and in the normalization we use its two simple roots are:

$$\alpha^1 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), \qquad \alpha^2 = \left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right). \tag{2.6}$$

Therefore the fundamental weights are

$$\mu^1 = \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right), \qquad \mu^2 = \left(\frac{1}{2}, \frac{-1}{2\sqrt{3}}\right). \tag{2.7}$$

There is one more positive root that is not simple $\alpha^3 = \alpha^1 + \alpha^2$ thus giving $\rho = (2, 0)$. A general representation of SU(3) is denoted by two positive integers $[r, s]$ and has a highest weight

$$\mu = r\mu^1 + s\mu^2 = \left(\frac{r+s}{2}, \frac{r-s}{2\sqrt{3}}\right). \tag{2.8}$$

We thus have:

$$Q^{\mathrm{SU}(3)}_{[r,s]} = \frac{(r+s)}{2}\frac{(r+s+4)}{2} + \frac{(r-s)^2}{12}. \tag{2.9}$$

Now the Cartan subalgebra of $\mathrm{SU}(3) \supset \mathrm{SU}(2) \times \mathrm{U}(1)$ is composed of the U(1) generator by which we mod out (denoted by K^8) and the Cartan generator K^3 of the SU(2). The U(1) charge under this K^8 is the y coordinate in weight space (x, y) while the x coordinate, which is always half integer, is the K^3 eigenvalue. Again, since we are looking for chiral primaries we must take the highest SU(2) weights in a given SU(2) representation.

We now use these results to compute (2.5) on the highest weight state of the SU(3) representation (with weight (2.8)). Since the charge of $H = \mathrm{U}(1)$ is the y entry in (2.8) and since the quadratic Casimir of a U(1) Lie algebra in a given representation is simply the square of the charge in that representation, we get:

$$Q^{\mathrm{U}(1)}_{[r,s]} = \left(\frac{r-s}{2\sqrt{3}}\right)^2. \tag{2.10}$$

We now use (2.9) and (2.10) to compute (2.5) on the highest weight, obtaining:

$$j(j+1) = \frac{1}{4}\frac{(r+s)}{2}\frac{(r+s+4)}{2}, \tag{2.11}$$

which is easily solved to give

$$j = \frac{r+s}{4}. \tag{2.12}$$

As explained above the x coordinate in weight space (x, y) is the unbroken SU(2) charge j' which serves as the R-symmetry in the spacetime superconformal algebra and therefore we can write for (2.8)

$$j_{ST} = j' = \frac{r+s}{2}. \tag{2.13}$$

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Comparing (2.13) and (2.12) we conclude that

$$h_{ST} = j = \frac{r+s}{4} = \frac{1}{2} j_{ST} \,, \tag{2.14}$$

i.e. these states are chiral.

Moreover let us now prove that there are no other chiral states coming from the same SU(3) representation. Candidate chiral states must be highest weights of an SU(2) representation, and it is sufficient to consider for every value of the U(1) charge the highest weight of the largest SU(2) representation. These SU(2) highest weight states are obtained from the highest weight μ of the SU(3) representation labeled by $[r,s]$ by $\mu - n\alpha_1$, with $n \leq r$, and $\mu - m\alpha_2$, with $m \leq s$.

Consider the state $\mu - n\alpha_1$. Using (2.8) and (2.6), its coordinates in weight space are determined to be:

$$\mu - n\alpha_1 = \left(\frac{r+s-n}{2}, \frac{r-s-3n}{2\sqrt{3}} \right) . \tag{2.15}$$

From the above expression we can read off that this is the highest weight of an SU(2) representation of spin $j' = \frac{r+s-n}{2}$, and that the U(1) charge is $\frac{r-s-3n}{2\sqrt{3}}$. Let us compute the following difference:

$$\begin{aligned} j(j+1) - \frac{j'}{2}\left(\frac{j'}{2}+1\right) &= \frac{1}{4}\left\{ \frac{(r+s)}{2}\frac{(r+s+4)}{2} + \frac{(r-s)^2}{12} - \frac{(r-s-3n)^2}{12} \right\} - \\ &\quad - \frac{(r+s-n)}{4}\frac{(r+s-n+4)}{4} \\ &= \frac{n}{4}(r+1-n) \geq \frac{n}{4} > 0 \,, \end{aligned} \tag{2.16}$$

thus proving that $h_{ST} = j > j'/2 = j_{ST}/2$ for a vertex operator built from any such state, i.e. they cannot be chiral. The same reasoning can be applied to the states $\mu - m\alpha_2$.

At this point we can prove the non chirality of the spacetime fields corresponding to vertex operators with only the χ oscillators. Since the fermions χ transform in the adjoint representation of SU(3) but do not carry any SL(2) index the spacetime scaling dimension of the field corresponding to such a vertex operator will be $h_{ST} = j+1$, and the component with largest SU(2) spin in the product (χU) has spacetime spin $j_{ST} = j' + 1$. So using the results (2.12) and (2.13) we can write

$$h_{ST} = j + 1 = \frac{1}{2}j' + 1 = \frac{(j'+1)}{2} + \frac{1}{2} = \frac{1}{2}j_{ST} + \frac{1}{2} > \frac{1}{2}j_{ST} \,, \tag{2.17}$$

i.e. these states are not chiral.

2.3 The $\mathrm{SO}(5)/\mathrm{SO}(3)$ coset CFT

The SO(5) algebra is not simply laced and has a length ratio of $\sqrt{2}$ between its two simple roots. Normalizing the highest root to length squared 1 we can choose

$$\alpha^1 = \left(\frac{1}{2}, \frac{-1}{2}\right), \qquad \alpha^2 = (0,1)\,. \tag{2.18}$$

Therefore the fundamental weights are

$$\mu^1 = \left(\frac{1}{2}, 0\right), \qquad \mu^2 = \left(\frac{1}{2}, \frac{1}{2}\right). \tag{2.19}$$

There are two more positive roots that are not simple $\alpha^3 = \alpha^1 + \alpha^2 = (1/2, 1/2)$ and $\alpha^4 = \alpha^1 + \alpha^3 = (1,0)$ thus giving $\rho = (2,1)$. A general representation of SO(5) is also denoted by two positive integers $[r,s]$ and has a highest weight

$$\mu = r\mu^1 + s\mu^2 = \left(\frac{r+s}{2}, \frac{s}{2}\right). \tag{2.20}$$

We therefore have:

$$Q^{\mathrm{SO}(5)}_{[r,s]} = \frac{(r+s)}{2}\frac{(r+s+4)}{2} + \frac{s(s+2)}{4}\,. \tag{2.21}$$

Now, since $\mathrm{SO}(5) \supset \mathrm{SO}(4) = \mathrm{SO}(3)_1 \times \mathrm{SO}(3)_2$ the Cartan subalgebra of SO(5) is composed of the two Cartan generators K^3_1 and K^3_2 of the two commuting SO(3)'s and we have to choose by which SO(3) to mod out. Choosing to mod out by the $\mathrm{SO}(3)_2$ subgroup whose Cartan generator gives the y coordinate in weight space (x,y), and again evaluating (2.5) on the highest weight state of the SO(5) algebra with weight (2.20) for which $Q^{\mathrm{SO}(3)_2}_{[r,s]} = \frac{s}{2}(\frac{s}{2}+1)$ we get again (2.11). In particular, we find also here chiral states for every spacetime R-charge $j_{ST} = j'$ coming from the NS sector of the worldsheet, built upon an SO(5) highest weight, and essentially the same reasoning as in the previous model demonstrates that these are the only chiral states in the NS sector.

3. The Ramond sector of the worldsheet CFT

So far we have analyzed the spectrum of chiral primary operators in the spacetime CFT arising from the NS sector of the worldsheet CFT. In this section we prove that the identification of chiral primaries done so far is complete, i.e. that there are no new chiral primaries arising from the Ramond sector.

A typical vertex operator in the Ramond sector can be written as:

$$e^{-\phi/2} S \Phi_{j,m} U_{r,j',\mathcal{Q}}\,, \tag{3.1}$$

where S is a spin-field (to be discussed below) and it is assumed that we have taken an appropriate linear combination such that the total SL(2) and unbroken SU(2) spins are fixed. Again, we are considering the lowest-dimensional operator with respect to the possible excitations as described generally in (2.1). With this taken into account, the condition for (3.1) to be a worldsheet primary of weight 1 is again (2.5), as for the NS vertex operators.

The quantum numbers carried by the spin-fields S were analyzed in [1]. We can write a general spin-field as:

$$S = e^{\frac{i}{2}(\epsilon_1 H_1 + \epsilon_2 H_2 + \epsilon_3 H_3 + \epsilon_0 \sqrt{2} H_0)} . \tag{3.2}$$

The scalars in the expression above can be obtained as follows. By bosonization, we get:

$$-\partial H_1 = \frac{2}{k}\psi^1\psi^2 , \qquad -\partial H_2 = \frac{2}{k'}\chi^1\chi^2 , \qquad -i\partial H_3 = \frac{1}{k}\psi^3\chi^3 . \tag{3.3}$$

Then the remaining scalar H_0 can be defined, for instance, through:

$$K^3 = \hat{K}^3 + i\partial H_2 + \frac{i}{\sqrt{2}}\partial H_0 . \tag{3.4}$$

Alternatively, it can be constructed by bosonization of a specific combination of the remaining fermions on G/H (see [1] for details).

A spin-field (3.2) has SL(2) spin $j = 1/2$ and SU(2) spin $j' = 1$. The total SL(2) and SU(2) spins of (3.1) are thus, respectively, $j \pm 1/2$ and $j' \pm 1$ or j'. The candidate chiral states in the R sector can be constructed using the same products of vertex operators used to build the chiral states from the NS sector, and taking the combinations with the spin-fields which lead to SL(2) spin $j - 1/2$ and SU(2) spin $j' + 1$. Indeed, we then have:

$$h_{ST} = j - \frac{1}{2} + 1 = j + \frac{1}{2} = \frac{j'}{2} + \frac{1}{2} = \frac{j'+1}{2} = \frac{j_{ST}}{2} , \tag{3.5}$$

where we used (2.12) and (2.13) which imply that $j = \frac{j'}{2}$ for such vertex operators.

We now have to explicitly check the BRST invariance of the vertex operators (3.1). The non-trivial part of this boils down to checking that the OPE of the worldsheet supercurrent with (3.1) has no $z^{-3/2}$ singular term.

Let us see in more detail the combination (3.1) giving rise to the desired spins. It is clear that if our aim is to write the vertex operator corresponding to a chiral operator in spacetime, this should be the highest weight of an SU(2) representation. We will thus concentrate on the $m' = j' + 1$ component of the vertex operator with total SU(2) spin $j' + 1$. This will be obtained using spin-fields which are highest weights of the spin 1 representation. We denote them by S^+ and they correspond to taking $\epsilon_2 = \epsilon_0 = +1$ in (3.2), as it can be determined from (3.4).

These spin-fields, labeled also by their SL(2) momentum, are given by:

$$S^+_{1/2} = e^{\frac{i}{2}(H_1+H_2+\epsilon_3 H_3+\sqrt{2}H_0)}\,, \qquad S^+_{-1/2} = -\epsilon_3 c_1 c_3 e^{\frac{i}{2}(-H_1+H_2-\epsilon_3 H_3+\sqrt{2}H_0)}, \tag{3.6}$$

where c_i are cocycle factors such that $c_i^2 = 1$ and $c_i c_j = -c_j c_i$. Note that we did not yet fix the GSO projection, which will amount to fixing the sign of ϵ_3. The prefactor $-\epsilon_3$ in $S^+_{-1/2}$ is determined by requiring that the two above spin-fields correctly transform as a spin 1/2 representation under the SL(2) currents given by:

$$J^3 = \hat{J}^3 + i\partial H_1\,, \qquad J^\pm = \hat{J}^\pm \mp c_1 c_3 e^{\pm iH_1}\left(e^{iH_3} - e^{-iH_3}\right). \tag{3.7}$$

The total spin $j-1/2$ combination can be written as:

$$\mathcal{V}_{j-\frac{1}{2},m} = S_{1/2}\Phi_{j,m-\frac{1}{2}} - S_{-1/2}\Phi_{j,m+\frac{1}{2}}\,. \tag{3.8}$$

Thus the candidate chiral state in the R sector is given by:

$$\begin{aligned}\mathcal{Y} = &\left(e^{\frac{i}{2}(H_1+H_2+\epsilon_3 H_3+\sqrt{2}H_0)}\Phi_{j,m-\frac{1}{2}} + \epsilon_3 c_1 c_3 e^{\frac{i}{2}(-H_1+H_2-\epsilon_3 H_3+\sqrt{2}H_0)}\Phi_{j,m+\frac{1}{2}}\right)\times \\ &\times U^{r,\mathcal{Q}}_{j'=2j,m'=2j}\,.\end{aligned} \tag{3.9}$$

There are three terms in the worldsheet supercurrent G which give rise to $z^{-3/2}$ singular terms in the OPE with $\mathcal{Y}$. The first term is the one which comes from the terms trilinear in the fermions, while the other two come from the terms including, respectively, the bosonic SL(2) and SU(2) currents:

$$\begin{aligned}G_f &= \frac{1}{\sqrt{k}}c_3\left[i\partial H_1\left(e^{iH_3}-e^{-iH_3}\right) + \frac{1}{2}\left(i\partial H_2 + \frac{i}{\sqrt{2}}\partial H_0\right)\left(e^{iH_3}+e^{-iH_3}\right)\right], \\ G_J &= \frac{1}{\sqrt{k}}\left[c_1 e^{iH_1}\hat{J}^- + c_1 e^{-iH_1}\hat{J}^+ + c_3\left(e^{iH_3}-e^{-iH_3}\right)\hat{J}^3\right], \\ G_K &= \frac{1}{2\sqrt{k}}c_3\left(e^{iH_3}+e^{-iH_3}\right)\hat{K}^3\,.\end{aligned} \tag{3.10}$$

We have neglected all terms which lead to less singular or regular behavior in the OPE with $\mathcal{Y}$. For a derivation of the full supercurrent G, see [1].

As a warm up, we can take the OPE of G with the spin-fields $S^+_{\pm 1/2}$. The only term contributing to the $z^{-3/2}$ singularity is G_f, and the singularity is avoided taking $\epsilon_3 = +1$. This determines the BRST invariant operators giving rise to the (global) spacetime $N=3$ supercharges, more precisely the highest weights of the spin 1 multiplets of them. This agrees with the result of [1], that the $N=3$ superalgebra in spacetime is recovered only for this GSO projection.[2]

Returning to $\mathcal{Y}$, since G_f only sees the spin-fields in $\mathcal{Y}$, its OPE is easily determined to be:

$$G_f(z)\mathcal{Y}(0) \sim z^{-3/2}\frac{1-\epsilon_3}{2}\mathcal{Y}'(0)\,, \tag{3.11}$$

[2]Note that there are some differences in the sign conventions with respect to [1], specifically, the signs in the bosonizations (3.3) here and [1, (3.7)] are opposite.

where $\mathcal{Y}'$ is:

$$\mathcal{Y}' = \frac{1}{\sqrt{k}} \Big(c_3 e^{\frac{i}{2}(H_1+H_2-\epsilon_3 H_3+\sqrt{2}H_0)} \Phi_{j,m-\frac{1}{2}} - \\ - \epsilon_3 c_1 e^{\frac{i}{2}(-H_1+H_2+\epsilon_3 H_3+\sqrt{2}H_0)} \Phi_{j,m+\frac{1}{2}} \Big) U^{r,\mathcal{Q}}_{j'=2j,m'=2j} \,. \tag{3.12}$$

Similarly one can work out the OPEs of the other pieces of G, giving:

$$\begin{aligned} G_J(z)\mathcal{Y}(0) &\sim z^{-3/2}\epsilon_3(j+1)\mathcal{Y}'(0)\,, \\ G_K(z)\mathcal{Y}(0) &\sim z^{-3/2} j\mathcal{Y}'(0)\,. \end{aligned} \tag{3.13}$$

The total contribution to the singularity of the OPE of G with $\mathcal{Y}$ is thus:

$$G(z)\mathcal{Y}(0) \sim z^{-3/2}\frac{1+\epsilon_3}{2}(2j+1)\mathcal{Y}'(0)\,. \tag{3.14}$$

This singularity is removed, and $\mathcal{Y}$ leads to a physical state, only if $\epsilon_3 = -1$. Since this is the opposite GSO projection to the one used in obtaining the $N = 3$ algebra we conclude that these operators are projected out and are not part of the physical spectrum.[3]

We have thus demonstrated that there are no operators coming from the R sector which are chiral states of the $N = 3$ superconformal spacetime theory.

4. Summary and comments on the worldsheet analysis

To summarize, we showed that every SU(3) (SO(5)) representation gives rise to a chiral state in spacetime, through the vertex operator (2.4) built using the highest weight of the representation. Moreover, we proved that these are the only vertex operators that correspond to chiral states in spacetime. We can read off the multiplicity of chiral states at each spacetime spin $j_{ST} = j'$ to be $2j'+1$ since we have such a state coming from each $[r,s]$ SU(3) (SO(5)) representation with $\frac{r+s}{2} = j'$, i.e. $[2j',0], [2j'-1,1], \ldots, [0,2j']$. Note that this multiplicity is not the degeneracy inside a spin j' representation. What we find are $2j'+1$ different SU(3) (SO(5)) representations each leading to a chiral primary in spacetime which is the highest weight ($m = j'$) of an $\mathrm{SU}(2) \subset \mathrm{SU}(3)$ (SO(5)) representation, itself with $2j'+1$ members.

The multiplicities of the chiral spectrum are the same for the two CFTs dual to the string backgrounds $AdS_3 \times \mathrm{SU}(3)/\mathrm{U}(1)$ and $AdS_3 \times \mathrm{SO}(5)/\mathrm{SO}(3)$. However one can easily get convinced that the non-chiral spectra are not identical. For example, in the $\mathrm{SO}(5)/\mathrm{SO}(3)$ model the state with lowest spacetime scaling dimension that is

[3] Note that performing this "wrong" GSO projection one ends up with a different spacetime theory with only $N = 1$ superconformal symmetry, and an additional affine SU(2) algebra that acts trivially on the supercharges [1]. Among the (now physical) Ramond sector operators are the lower components of the SU(2) currents with respect to this spacetime $N = 1$ supersymmetry.

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a scalar of the SU(2) R-symmetry satisfies $h_{ST}(h_{ST}+1) = 1/8$. Such states do not exist in the SU(3)/ U(1) spacetime CFT where the neutral state of lowest dimension satisfies $h_{ST}(h_{ST}+1) = 1/4$. We therefore conclude that the agreement of the chiral spectrum suggests that the two models are sitting at two different points of the same moduli space.

Indeed we can identify three marginal deformations of the dual CFT. The two coset models discussed here are "geometrically rigid", as opposed to [3, 8] where, for instance, radii of various U(1)'s appear as NS moduli of the worldsheet sigma model. However, we can identify three marginal deformations that come from the R sector of the worldsheet CFT. We start from the three $j_{ST} = 1$ chiral operators with weight $h_{ST} = 1/2$. Taking the commutator of these with the spacetime supercharges which have the same quantum numbers we can pick out the SU(2) scalar from that product. This operator has a weight and R-charge obeying $h_{ST} = 1$, $j_{ST} = 0$ and is thus a marginal deformation of the $N = 3$ superconformal algebra. Furthermore it is an exactly marginal deformation because it is the upper component of a chiral operator.[4]

A last remark is that the unitarity bound [7, 16] $-1/2 < j < \frac{k-1}{2}$ on the SL(2) spin imposes a bound $j' < k'/4 - 1$ on the SU(2) spin which is generically stricter than the one required for unitarity of the G/H CFT.

5. Comments about the nature of the spacetime CFTs

Having computed the untwisted part of the chiral spectrum of the spacetime CFTs dual to the string backgrounds $AdS_3 \times \mathrm{SU}(3)/\mathrm{U}(1)$ and $AdS_3 \times \mathrm{SO}(5)/\mathrm{SO}(3)$, which turns out to be the same, we will now draw some conclusions about the nature of these dual CFTs.

Following the general result of [2] we know that the structure of the spectrum of the spacetime dual CFT is the same as that of a symmetric product CFT $(\mathcal{M}_{6k})^p/S_p$. In some cases [3, 8] one can go further and show that the building block model $\mathcal{M}_{6k}$ itself is expected to be in the moduli space of a symmetric product CFT. Specifically, for the background $AdS_3 \times S^3 \times T^4$ the "untwisted" part of the dual CFT (in the language of [2]) was argued to be in the moduli space of the small $N = 4$ CFT on $(T^4)^k/S_k$ (where k is the level of both the SL(2) and SU(2) WZW models), and for the background $AdS_3 \times S^3 \times S^3 \times S^1$ the "untwisted" part of the dual CFT was argued to be in the moduli space of the large $N = 4$ CFT on $(\mathcal{M}_3)^{k'}/S_{k'}$ (where $\mathcal{M}_3$ is a $c = 3$ theory of one scalar and its four fermionic superpartners, and $k' = 2k$ is the level of both SU(2) WZW models). This property is related to

[4]Strictly speaking, we considered in the above paragraph only "single particle" marginal deformations. In this $N = 3$ set up, where the smallest chiral state has $h_{ST} = 1/4$, it should be possible (if the statistics allows it) to consider a chiral two-particle state with $h_{ST} = 1/2$. Then one would proceed as above to build an exactly marginal deformation. Of course these states cannot be seen from the perturbative worldsheet perspective.

U-duality and the simple brane picture associated with such high supersymmetry examples. As discussed in [2, section 6] this led to the expectation that the full dual CFT in these two cases is also in the moduli space of $(T^4)^{kp}/S_{kp}$ and $(\mathcal{M}_3)^{k'p}/S_{k'p}$, respectively.

One might therefore ask whether a similar thing will happen in our case, i.e. is the building block $\mathcal{M}_{6k}$ also in the moduli space of a symmetric product CFT? If so this would mean that the full spacetime CFT will be in the moduli space of a symmetric product of (an integer of order kp) copies of a building block with some fixed and small central charge. This expectation seems to "make sense arithmetically" also because in [1] it was shown that the central charge of the spacetime CFT in the two models is $\tilde{c} = \frac{3}{2}\tilde{k} = \frac{3}{2}k'p$ where k' is the level of the affine $\hat{G}$, thus suggesting a building block CFT with central charge 3/2 and an $N = 3$ superconformal symmetry. The unique candidate for such a building block was introduced in [11] and is composed of a scalar X compactified on a circle at the self dual radius, or an $\mathrm{SU}(2)_1$ WZW model, together with a free fermion uncharged under this SU(2). This is the only such model since, seen as an $N = 2$ minimal model, it corresponds to the unique one with $c = 3/2$. Denoting this building block theory by $\mathcal{C}$ one might thus look at the symmetric product $\mathcal{C}^{k'}/S_{k'}$ (which has the right central charge $\frac{3}{2}k'$ and which inherits the $N = 3$ superconformal symmetry in the form of the diagonal algebra) as a candidate CFT to be in the moduli space of $\mathcal{M}_{6k}$.

It turns out however that unlike the $N = 4$ examples, $\mathcal{M}_{6k=\frac{3}{2}k'}$ cannot be itself in the moduli space of some symmetric product of a smaller building block. The reason for that is that the chiral spectrum has a multiplicity of $2j' + 1$ chiral states for every R-charge j', while in any symmetric product the multiplicity of chiral states corresponding to "single particle states" at every energy (charge) level is bounded from above by the total number of chiral operators in the building block theory.

To see this we adapt here the results of [2, section 5] to show that there is a one-to-one correspondence between chiral states in a building block theory $\mathcal{M}$ (including the identity) and chiral states in each of the $\mathbb{Z}_N$ twisted sectors of the symmetric product $\mathcal{M}^{k'}/S_{k'}$.[5] Moreover this correspondence is such that the chiral spectrum of the $\mathbb{Z}_N$ twisted sector is a shifted version of the chiral spectrum of $\mathcal{M}$ itself. Specifically one can show [2] that a chiral state in $\mathcal{M}$ with R-charge R and weight $R/2$ leads to a unique chiral state in $\mathcal{M}^{k'}/S_{k'}$ coming from the $\mathbb{Z}_N$ twisted sector with weight h^N and R-charge R^N that satisfy:

$$h^N = \frac{1}{2}R + \frac{c}{12}(N-1)\,, \qquad R^N = R + \frac{c}{6}(N-1)\,. \tag{5.1}$$

This means that as N increases by 1 the spectrum of chiral R-charges is shifted by $c/6$.

[5] Recall that the spectrum of the $\mathbb{Z}_N$ twisted sectors of $\mathcal{M}^{k'}/S_{k'}$ corresponds to operators associated with single particle states, on which we focus in this paper.

Since the step by which the weights are translated is constant, it is straightforward to get convinced that indeed the multiplicity of chiral states at every R-charge value is bounded from above by the total number of chiral operators in $\mathcal{M}$.

One can actually compute directly the multiplicities of the chiral operators in $\mathcal{C}^{k'}/S_{k'}$. This is done by first writing the extended partition function of the model $\mathcal{C}$, which is the product of the extended partition function of an $\mathrm{SU}(2)_1$ WZW model and that of a free fermion — the Ising model. Then using the general theory of cyclic orbifold CFTs [17, 18] (see also [2, appendix B]) one can identify the weight and R-charge of states in the $\mathbb{Z}_N$ twisted sectors, and find the chiral states for every such sector. The result is that for every $\mathbb{Z}_N$ sector with N odd there are exactly two chiral operators with R-charge $j_{ST} = \frac{N\pm1}{4}$. We thus conclude that for any given R-charge j_{ST} there are exactly two chiral operators that come from the $\mathbb{Z}_N$ sectors with $N = 4j_{ST} \pm 1$. This result agrees of course with that of the arguments above. Indeed there are exactly two chiral operators in $\mathcal{C}$ which are the identity and the $j_{ST} = 1/2$ operator $e^{\frac{i}{\sqrt{2}}X}$.

6. Conclusions

Summarizing, we computed the chiral spectrum of the untwisted sector of the space-time CFTs dual to the two string backgrounds $AdS_3 \times \mathrm{SU}(3)/\mathrm{U}(1)$ and $AdS_3 \times \mathrm{SO}(5)/\mathrm{SO}(3)$ and showed that in both cases for every R charge j_{ST} there are $2j_{ST}+1$ chiral operators originating from the NS sector on the worldsheet. The dual CFT is expected to be $(\mathcal{M}_{6k})^p/S_p$ [2]. Inspired by the $N = 4$ cases where the $c = 6k$ building block theory itself was argued to be in the moduli space of a symmetric product, we asked whether or not this is true also for the two $N = 3$ models at hand, and showed that the answer was negative.

Actually, if one looks at $N = 2$ cases this phenomenon where the building block theory itself is in the moduli space of a symmetric product generically fails due to simple algebraic considerations [2, 19]. Consider for example the case of a background of the form $AdS_3 \times S^1 \times \mathcal{M}_n$ where $\mathcal{M}_n$ is the $N = 2$ minimal model with central charge $c_n = 3 - 6/n$. On the worldsheet this is realized as the sigma model $\mathrm{SL}(2)_k \times \mathrm{U}(1) \times \mathcal{M}_n$. Solving for criticality of the worldsheet theory one finds that $k = 1 - \frac{1}{n+1}$. This is generally not an integer so trivially the CFT $\mathcal{M}_{\frac{6n}{n+1}}$ cannot be in the moduli space of a symmetric product, even arithmetically, because its central charge is not linearly "quantized."

We can thus view the $N = 3$ coset examples as intermediate cases between the most symmetric $N = 4$ case for which the symmetric product ansatz for $\mathcal{M}_{6k}$ works, and the generic $N = 2$ cases where such an ansatz does not even make sense. Indeed in the $N = 3$ case there was a sensible and unique candidate, and its spectrum did reproduce some of the features calculated in string theory, such as the existence of a chiral operator for every R-charge j_{ST}, however the bottom line is negative. Note also that the arguments of [6, 20, 21] in favor of a spacetime CFT based on a more general symmetric product in the $N = 4$ case rely on U-duality and the simple brane picture of these highly symmetric cases that possibly do not apply here. Another possibility is that at some other points in the moduli space of the symmetric product some multiparticle states are marginally bound and can thus be matched with the single particle states computed from string theory. It is more likely however that the dual CFT is simply not a symmetric product.

Finally we should comment that there is another model that shares the same spacetime symmetry structure, namely $AdS_3 \times (\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1))/\mathbb{Z}_2$ [12] which is a $\mathbb{Z}_2$ orbifold of the model with large $N = 4$ superconformal symmetry [8]. A natural question that arises is the connection between this model and the two models discussed in this paper, specifically whether or not they belong to the same moduli space. The results of this paper lead us to expect that the spacetime theory of this string background is not in the same moduli space as the two spacetime CFTs discussed in this paper. The reason is that $AdS_3 \times (\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1))/\mathbb{Z}_2$ is an orbifold of a theory which has the property that $\mathcal{M}_{6k}$ is in the moduli space of a symmetric product, and thus it might also share this property, while the CFTs dual to $AdS_3 \times \mathrm{SU}(3)/\mathrm{U}(1)$ and $AdS_3 \times \mathrm{SO}(5)/\mathrm{SO}(3)$ where shown not to possess this property.

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